How to Gamble If You Must: Inequalities for Stochastic Processes. by Lester E. Dubins; Leonard J. Savage
Review by: Morris H. DeGroot
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even poker. While appropriate examples may necessarily strain the subject matter field to fit the level of exposition, at least the intelligent layman could be made aware that there is more to the subject than replacing spark plugs in cylinders (P. 87).

In spite of this, the book is to be highly recommended to those who want to learn "how to count without counting" (the book's subtitle), as well as to those whose job it is to communicate how much mathematics can be developed with a minimum of assumptions and techniques. At the price, no one even faintly interested in mathematics should be without this book.


EUGENE D. NICHOLS, The Florida State University

This is a brief treatment, in a programmed style, of some elementary concepts of logic, statistics, and probability. The topics included are the role of premises and inferences in a logical argument, some notation of symbolic logic, truth tables, fundamental notions of probability (including the probability of combined events), Bayes' rule, permutations, combinations, the binomial distribution, expected value, variance, and standard deviation. A little more than two chapters are devoted to problems which make use of some of the fundamental concepts of probability.

The program has undergone unusually extensive testing, which is reflected in rather carefully constructed responses. The style of the program varies in that sometimes a student is called upon to supply a missing symbol, word, or phrase; at other times he is presented with several choices, only one of which is correct. In case of an incorrect choice, the student is directed to a page where he is led to see his error. He is then directed to return to the original page, reread the question or problem and make another choice.

The author suggests that 20 hours should be a reasonable amount of time in which to complete the program; however, as is the case with all programs, some students need more time and others can complete it in less time.

Each chapter is preceded by a brief preview of what is to be found in the chapter. At strategic points, brief summary frames are included, serving the useful purpose of providing the student with a review of what has preceded. At the conclusion of each chapter is supplied a posttest, followed by a set of exercises (to be used if further review is desired).

In introducing a few of the fundamental notions of statistics, the author employs a rather unusual notation: a capital letter denotes the negation of a statement denoted by the corresponding lower case letter (instead of the usual ~p), AB means both A and B (instead of the usual A ∩ B), and A + B means either A or B or both A and B (instead of the usual A ∨ B, ∨ denoting the inclusive or).

It is assumed that the reader is able to make, on his own, the connection between the conjunction and the intersection of sets and between the disjunction and the union of sets. That these are related is merely hinted in footnotes.

The style of writing throughout is informal, and no words are wasted.

Thus, this is a good brief programmed text for those interested in the topics listed in the first paragraph.


MORRIS H. DEGROOT, Carnegie Institute of Technology

One might expect a book entitled How to Gamble If You Must to be subtitled Mathematics for the Million, but the book under review here is, in fact, subtitled
Inequalities for Stochastic Processes. How do title and subtitle fit together? In the first paragraph of the first chapter, Dubins and Savage present the following “fantasy” to exemplify the type of problem they attack:

"Imagine yourself at a casino with $1,000. For some reason, you desperately need $10,000 by morning; anything less is worth nothing for your purpose. What ought you do? The only thing possible is to gamble away your last cent, if need be, in an attempt to reach the target sum of $10,000. There may be a moment of moral confusion and discouragement, for who has not been taught how wrong and futile it is to gamble, especially when short of funds? Yet, gamble you must or forego all chance of the great purpose that can be achieved only at the price of $10,000 payable at dawn. The question is how to play, not whether."

In the last paragraph of the last chapter, they state:

"... A solution of gambler's problems implies an inequality for stochastic processes. The search for a bound on some probability associated with a stochastic process or class of processes can sometimes usefully be reformulated as a gambler's problem. In short, much of the mathematical essence of a theory of gambling resides in the discovery and demonstration of sharp inequalities that pertain to interesting classes of stochastic processes."

It is seen from the first quotation given above that the title How to Gamble If You Must is not only attractive but, within a limited context, appropriate as well. However, it is made clear on the dust jacket that this is a "technical mathematical monograph" and that it "is addressed to research mathematicians." The first line of the preface states that this work forms "a chapter in the pure mathematics of probability." These statements, together with the subtitle, serve the same purpose as the standard warning: Keep out of the reach of children. Here, however, the consequences to the mathematically immature individual who reaches for this book and takes excessive doses, are probably no more serious than a headache and the loss of $12.75. Because it gives an intense, detailed, mathematical treatment to a highly specialized class of problems, the book appears to be of somewhat limited value to gamblers. Thus, its spirit, approach and content are different from those of E. O. Thorp in "A favorable strategy for twenty-one," Proc. Nat. Acad. Sci., Vol. 47 (1961) pp. 110-112, and Beat the Dealer, Blaisdell, New York, 1962 (and yet it is mildly surprising that Dubins and Savage do not mention this work, surely one of the more interesting mathematical contributions to gambling in recent years).

Once it is understood that the book is to be viewed (and reviewed) through the eyes of "pure mathematicians," it is readily seen to be an admirable achievement. It is filled with striking, original results obtained by Dubins and Savage, and some of their friends. They have formulated, solved, organized, and described in a concise, rigorous and sophisticated manner a class of problems whose mathematical history goes back to DeMoivre and Bernoulli. Since these problems stem from highly idealized gambling situations, they can be naturally presented in the language of gambling; and some of the more difficult mathematical results have fanciful descriptions in gambling terms. As a result, parts of the book have colorful tones not often found in advanced mathematical monographs.

In the first of the book's twelve chapters, Dubins and Savage give an excellent, brief introduction and summary.

Chapter 2 is devoted to the formulation of the abstract gambler's problem. In order to avoid measurability difficulties that become especially bothersome in sequential decision problems, the authors depart from the usual definition of probability as a countably additive measure defined on a sigma-field of subsets of the underlying space. For them, a probability measure, or gamble, assigns a probability to every subset of the space but need only be finitely additive. (One consequence of this
usage is that many examples and remarks involve *diffuse* gambles that assign probability 1 to the set of all integers but probability 0 to every finite set of integers.) The basic discussion and results of this chapter are relevant not only to the special gambler’s problems treated later in the book, but to a broad class of sequential decision problems.

In Chapter 3 the existence and properties of optimal strategies are discussed. In parts of this chapter the abstract formalism seems to take over and entangle the reader in relatively unproductive considerations. For example, there is a mathematical study of the relationship between a gambler stopping at some point with a certain fortune and his continuing indefinitely from that point using a strategy that will keep his fortune forever unchanged. In the course of this study, the authors give a theorem that lists eight different ways of saying that under a certain strategy all the gambles are sure things, and consequently, the gambler’s fortune will change in a fixed, predetermined way.

A gambling house is defined in terms of the gambles that are available when the gambler’s fortune is at each of its possible values. In Chapter 4 a particular type of gambling house, called a casino, is introduced and studied.

Chapter 5 is concerned with the special casino where a gambler wins an amount \( s \) with probability \( u < 1/2 \) and loses \( s \) with probability \( 1-u \), and he can, at any stage, risk as much of his fortune as he wishes. The gambler’s problem is to maximize the probability of reaching a fixed fortune \( f \) before going broke. In this context, bold play is defined to be the strategy where the gambler risks at each stage either his entire fortune or, whenever possible, just enough so that his fortune will be \( f \) if he wins. Dubins and Savage show that bold play is optimal and that typically there are other optimal strategies (such as following what would be bold play if the goal were \( f/2 \) and then, if this fortune is reached, risking it all on one last gamble). These, and the related results of this chapter and the next, are highlights of the book.

In Chapter 6 the above results are extended to a more general type of casino. It is shown, for example, that if a gambler is allowed at each stage to divide the amount he bets equally among as many numbers of a roulette table as he wishes, he should still play boldly and should bet on only one number at a time. One of several open questions indexed at the end of the book is whether this remains true when he is allowed to stake different amounts on different numbers.

Chapters 7 and 8 deal with the properties of some general types of gambling houses. Chapters 9, 10, and 11 deal with measures of subfairness for casinos and with optimal strategies in casinos where favorable or fair gambles are available. All of these chapters are liberally sprinkled with interesting results.

In the final chapter, Dubins and Savage illustrate how the concept of a gambler’s fortune is of sufficient generality for their theory of gambling to include much, or even all, of dynamic programming and Bayesian decision theory. It remains to be seen whether the results presented in this book will help solve some of the significant problems in these areas.

The book is strongly recommended for those with sufficient training and time. Indeed, those with the training should find the time.


**Howard G. Schaller, Indiana University**

This volume presents the papers from the second conference sponsored by CORA (Committee on Regional Accounts). The papers from the first conference were published in Werner Hochwold (ed.), *Design of Regional Accounts* (Baltimore: The