Disease Eradication: Private versus Public Vaccination

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Despite the fact that infectious diseases remain one of the major causes of morbidity and mortality in the world, especially in developing countries, economic research on policies aimed at limiting their occurrence has been limited. A major technology aimed at limiting such diseases has been vaccines. Although the introduction of a vaccine usually produces a sharp drop in the occurrence of a disease, the eradication of vaccine-preventable diseases predicted by many at the time of these inventions has never been achieved. Of the roughly 40 vaccines on the market, only the smallpox vaccine has been successful in eradication. Diseases such as measles, tuberculosis, and different types of influenza persist, despite explicit governmental efforts to eradicate them, and recent attempts to develop a vaccine against HIV or AIDS raise important questions about the causes behind these difficulties.

This paper provides a positive analysis of the forces that make disease eradication difficult, whether attempted through the public or the private sector. Section I studies the equilibrium determinants of the demand for vaccines and the implied disease dynamics in private markets. A central interaction stressed is that between the extent of disease, which is lowered by the demand for vaccines, and the demand for vaccines, which is increased by the extent of disease. We show a very general result on the difficulty of private markets to eradicate disease, regardless of the market structure under which vaccines are produced. In particular, price reductions achieved by competition do not imply eradication. Our simple, but general, result stems from the fact that as the disease disappears, so too does the demand for vaccines, subsequently allowing the disease to return.

Section II analyzes the difficulties of achieving disease eradication through standard public health interventions, such as price subsidies and mandatory vaccination programs aimed at stimulating demand. Our analysis implies that these policies are limited in their ability to increase demand, and hence to achieve eradication. Price subsidies alone will not bring about eradication for the same reasons that price reductions through increased competition will not. Both price subsidies and mandatory vaccination programs are limited in their ability to achieve eradication because the increase in demand of individuals covered by the programs lowers the incentive to vaccinate for those outside the program, due to any reduced prevalence brought about by a program. This brings into doubt classic justifications of Pigouvian subsidies aimed at solving the underprovision of vaccines due to their positive external effects. This is so because the price elasticity of demand is lower the more demand responds to disease occurrence, with the extreme case of total demand being inelastic to subsidies. Nevertheless, we show that price subsidies may make it potentially profitable for a monopolist to eradicate the disease. However, a vaccine monopolist faces a nonstandard dynamic incentive to increase markups. For if the vaccine eradicates the disease, the demand for the monopolist’s product is eradicated as well.

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Finally, Section III concludes by considering the normative aspects of eradication. We argue that the important welfare effects are dynamic and that a deficit-financed eradication program may improve welfare by making future generations pay current ones for the benefits of disease elimination.

I. Disease Eradication in Private Markets

Individuals are classified into four health categories: susceptible, infected, immune through recovery, and being outside the system, with the fractions in the first three categories denoted $S_t$, $I_t$, and $R_t$ at time $t$. We refer to the fraction infected in the population, $I_t$, as the prevalence of the disease. A future path of prevalence is denoted $I'[s \geq t]$ and a future path of prices is denoted $p'[s \geq t]$, with the instantaneous demand for vaccines at time $t$ for two such paths denoted $D(I', p')$. We denote by $b$ and $m$ the birth and mortality rates into and out of the system, respectively, and by $w$ the rate at which infected individuals are withdrawn naturally from infection into immunity. The changes over time in the health of the population are determined by

$$
\begin{align*}
\frac{dS_t}{dt} &= b[1 - D(I', p')] - \beta S_t I_t - mS_t, \\
\frac{dI_t}{dt} &= \beta S_t I_t - (w + m)I_t, \\
\frac{dR_t}{dt} &= bD(I', p') + wI_t - mR_t.
\end{align*}
$$

The change in the fraction of susceptible individuals is due to the entry of newborn individuals who do not vaccinate. Exits are due to new infections and nondisease-related mortality. The change in the prevalence is due to the entry of new infections, while exits are due to immunity and infection-related mortality. New infections are caused by contact between susceptible and infected individuals under random matching, as in the term $\beta S_t I_t$, where $\beta$ denotes the probability of transmission conditional on a potentially infective match. The change in the fraction of recovered immune individuals is due to the entry of newborn individuals who vaccinate, as well as those individuals recovering from infection, with exits due to nondisease-related mortality.

It follows directly that the prevalence rises over time whenever

$$
\frac{dI_t}{dt} \geq 0 \iff \frac{1}{w + m} \beta S_t \geq 1.
$$

The factor $\beta S_t$ is the rate at which infected individuals infect, and the factor $1/(w + m)$ is the average time of infection. For the infected stock to grow, it is therefore required that the average number of secondary infections by an infected individual be above unity, so that an infected individual more than replaces himself among the infected. When there are only susceptible individuals, which is the relevant case when a disease is to be eradicated, the secondary infections of a new infection are denoted $R_0 = \beta/(w + m)$, so that the disease can take off in a completely susceptible population only when this rate is above unity.

These prevalence dynamics, coupled with rational demand for vaccines, provide for powerful forces that make it difficult for private markets to achieve eradication. We call the demand for vaccines prevalence dependent if, when prices are positive in the future, demand vanishes when prevalence is low enough. That is, for any strictly positive price path $p'$, there is a prevalence path $I'(p')$ below which demand vanishes: $D(I', p') = 0$ for all $I' \leq I'(p')$. The Appendix demonstrates in more detail the weak assumptions on individual demand behavior that are sufficient to guarantee a prevalence-dependent demand. It stems simply from the fact that the benefits of vaccination are not large enough when prevalence levels are low enough. If demand is prevalence dependent, and if the prevalence goes to zero for any future prices, there must be a time $t_0$ after which the prevalence is driven down so low as to generate zero demand. As no one vaccinates after $t_0$, the population becomes increasingly susceptible. However, when an infection can regenerate itself in a susceptible population (i.e., when $R_0 \geq 1$), this implies that the prevalence

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2 Since we are not focusing on the effects of population change, we assume throughout that there is no infection-induced mortality, and that the birth rate equals the mortality rate, $b = m$. 
increases again, making eradication infeasible. In other words, the disease cannot be eradicated under positive prices when $R_0 \geq 1$.

As this argument holds for any prices, it implies that, regardless of the market structure in which vaccines are produced, the disease is not eradicated, since prices presumably are above costs in the long run for any market structure. In particular, although a vaccine monopolist is faced with a problem similar to that of using an exhaustible resource, the resource (prevalence) never will be exhausted. Naturally, if competition drives prices down to reflect minimum average costs of production, then eradication is not achieved under this market structure either. Interestingly, this argument not only is robust to the type of market structure, but also to many forms of expectations: it is true under myopic as well as rationally formed expectations. The general difficulty with eradication thus comes from the demand side of the vaccine market, as opposed to the supply side.3

II. Disease Eradication Through Public Health Interventions

The previous section attempted to explain the observed difficulty of private markets to achieve eradication. This section discusses the forces that explain the same difficulty for public health programs. For expositional purposes, we consider the effects of such interventions on the steady state of the system outlined above. In the steady state, the fraction of individuals in each of the three health states remains constant over time at levels $(S, I, R)$, now denoted without time indices. Denote by $D(I, p)$ the demand under a constant future prevalence path at level $I$, where $D_p \leq 0$ and $D_I \geq 0$. The positive sign of $D_I$ we refer to as the prevalence response of demand. As discussed in the Appendix, this is simply the result of an increased benefit of vaccination as prevalence rises. We also show in the Appendix that for each stationary price, there is a unique4 steady state of prevalence denoted $I(p)$, which is increasing in price and is locally stable. Therefore, contrary to many aggregate dynamics of economic systems, no cycles can occur in this system even with myopic demand: vaccination-induced cycles only can occur with a lagged prevalence response of demand.

A. Public Price Subsidies

Using the unique and positive relationship between prevalence and price, vaccine demand can be written as a function of price alone, as $\bar{D}(p) = D(I(p), p)$. The total effect of a price increase on this demand then consists of not only the standard direct negative effect but also the indirect and positive effect through the increased prevalence

$$\bar{D}_p = D_p + D_I \cdot I_p.$$ 

The indirect positive effect is dependent upon the degree to which prevalence rises with price. This effect, in turn, is negatively related to the prevalence response in demand, as can be seen by the following steady-state relationship derived in the Appendix:

$$I_p = -\frac{1}{D_I + \frac{w + m}{b}} \cdot D_p.$$ 

As price increases, demand decreases, causing prevalence to increase. This, in turn, creates a counteracting feedback in the demand, which limits the impact that price has on prevalence. The more prevalence responsive is demand, the larger is this counteracting effect. Consequently, the total effect of price on demand falls with the prevalence response of demand, as can be seen by substituting in $I_p$

$$D_p = \left(\frac{1}{1 + \frac{b}{w + m} D_I}\right) \cdot D_p.$$ 


4 We ignore the steady state $(S, I, R) = (1, 0, 0)$ with zero prevalence.
The total price effect discounts the partial effect by a factor that falls with the prevalence response of demand. In sum, the larger is the prevalence response of demand, the less it responds to price.

The implied reduction of price elasticities has important implications for interventions traditionally justified by economists as resolving the under provision problem of vaccines due to their positive external effects. Under a prevalence-responsive demand, the small price elasticities may limit the effects of Pigouvian price subsidies.

B. Prevalence-Responsive Subsidies

Governments usually respond to increased prevalence by expanding the extent and amount of subsidies during disease outbreaks (see Philipson and Posner, 1993; Philipson, 1996; Geoffard and Philipson, 1996). When subsidies vary with prevalence, as in \( s(I) \), we call them countercyclical or procyclical, depending on whether the slope of \( s(I) \) is negative or positive, respectively. Without a doubt, the majority of public-sector subsidy programs, whether international, national, or subnational, are procyclical. The total steady-state demand function with such subsidies is

\[
D^* (I, p) = D (I, p - s(I)).
\]

The total prevalence response of demand is then

\[
\frac{dD^*}{dI} = D_I - sD_p.
\]

An increase in prevalence affects subsidized demand directly, but it also affects it indirectly through the subsidy response, which induces a price reduction under procyclical policy, as opposed to a price increase under countercyclical policy. In the Appendix, we show that the larger is the prevalence response of demand, the lower is the steady-state prevalence, in the sense that \( I \) is lower whenever \( D_I \) is uniformly larger. This directly implies, therefore, that a procyclical subsidy policy lowers steady-state prevalence more than a countercyclical policy. Furthermore, since the impact of price on demand is smaller when the prevalence response of demand is larger, this implies that the effects of a less competitive market structure are mitigated by procyclical subsidies. The more procyclical are subsidies, the less the market structure affects prevalence.

C. Subsidized Market Eradication

One interesting aspect of subsidies is that they open up the possibility that it may be profitable for suppliers to eradicate the disease—something that was never true in a private market.

For a competitive market facing a stationary subsidy \( s \), the market eradicates the disease only if the subsidy covers the minimum average cost of production. On the other hand, a monopoly producer of vaccines faces an unconventional incentive to keep the disease alive: if the disease is eradicated, so is the demand for the monopolist’s product. Consider a monopolist which faces constant marginal costs \( c \). Clearly, the monopoly price never will be below the subsidy level since demand is completely price inelastic at such a price. If the monopoly price equals the subsidy, the vaccine is free with a universal demand \( D(0) = 1 \), which is assumed to lead to eradication after a length of time \( T \). Profits \( \Pi_E \) under such subsidized eradication are

\[
\Pi_E = \int_{t=0}^{T} (s - c) D(0) e^{-\delta t} dt,
\]

where \( \delta \) is the discount rate. If the monopolist price is above the subsidy level, the disease is not eradicated and the corresponding profits \( \Pi_N \) under price \( p \) are

\[
\Pi_N = \int_{t=0}^{\infty} (p - c) D(p - s) e^{-\delta t} dt.
\]

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5 These Pigouvian subsidies are discussed in most standard treatments of public finance or health economics, such as Joseph Stiglitz (1988), Victor Fuchs (1989), and Charles Phelps (1992). See also Thomas Schelling (1978), Dagobert Brito et al. (1991), and Christopher Avery et al. (1995).
Comparing the two, we have that eradication is profitable (i.e., \( \Pi_E \geq \Pi_N \)) if

\[
\int_{t=0}^{T} \left[ (s - c) \hat{D}(0) \right. \\
- (p - c) \hat{D}(p - s) \left. \right] e^{-bt} dt
\geq \int_{t=-T}^{\infty} (p - c) \hat{D}(p - s) e^{-bt} dt.
\]

This says that increased profits earned on the high level of demand needed to induce eradication are larger than the loss in future profits due to the fact that the product of the monopolist has become valueless. However, the important point here is that eradication is less likely to be profitable the more responsive demand is to prevalence. This is so because eradication is less profitable the less demand responds to price and, as discussed, \( \hat{D}_p \) falls with \( D_t \). When demand is price inelastic, it never pays to eradicate because the monopolist earns a loss both before and after the disease is eradicated: a loss after eradication (as discussed) because the product is valueless, and a loss before eradication because raising the price will increase current profits. It is straightforward to show that the condition above, \( \Pi_E \geq \Pi_N \), reduces to \((s - c)(1 - e^{-BT}) \geq (p - c)D(p - s)\). This further implies that the less future profits are discounted (i.e., the larger is \( \delta \)), the more eliminating the demand for the product in the future matters, and thus the less likely it is for eradication to be profitable. This effect of the interest rate is important because of a contrasting impact it has on social welfare, which will be discussed in the next section. In sum, if demand is highly responsive to prevalence or if discounting is moderate, subsidized eradication is not profitable.6

D. Mandatory Vaccination

Virtually all observed mandatory vaccination programs are partial—they do not cover whole populations or even whole age groups. Therefore, private decisions to vaccinate outside of public programs remain an important component of the total demand for vaccination. The total demand when a public program covers a fraction \( f \) of the population is given by

\[
D_T(I, p, f) = f + (1 - f)D(I, p).
\]

The first term is the mandatory demand in the program, while the second term is the private demand outside the program. Partial mandatory programs crowd out the private demand for vaccination outside the program, in the sense that some individuals would vaccinate in the absence of the program, but would not vaccinate in its presence.7 More precisely, the marginal effect of increased public coverage demand is

\[
\frac{dD_T}{df} = [1 - D] + (1 - f)D_t I_f.
\]

The first term is the direct positive effect resulting from the increased public coverage of individuals who otherwise would not have vaccinated. The second term is the indirect negative effect on private demand for vaccination by individuals not covered by the public program. The latter effect is increasing in the prevalence response of demand, thereby limiting the ability of mandatory vaccination programs to increase vaccination.

To illustrate, consider the private demand given by the estimable discrete-choice model involving an index specification

\[
D(I, p; \theta) = g(\theta_0 + \theta_p p + \theta_I I),
\]

where the demand parameters \( (\theta_0, \theta_p, \theta_I) \) may be estimated by standard methods of discrete-

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6 Subsidized suppliers, rather than demanders, may not be prevalence elastic under so-called supplier-induced demand. Supplier subsidization was undertaken in England in 1990 when general practitioners received bonuses if they achieved prespecified immunization targets for their patients. The fraction who achieved the targets increased from 55% at the start to 85% at the end of the program in 1992 (Principal Medical Officer, Department of Health, England, 1994). This policy raises the interesting question of whether health activities with positive external effects may be efficiently provided by supplier-induced demand.

7 This crowding-out effect is one possible interpretation of the relatively low pre-school vaccination rates in the United States, given the mandatory vaccination required in public schools.
choice econometrics. The total demand function is then

\[ D_T(p; \theta) = f + (1 - f)D(I(f, p, \theta), p; \theta), \]

where \( I(f, p; \theta) \) is the steady-state prevalence. To illustrate the crowding-out effects of this type of demand, Figure 1 plots the total demand curve \( D_T \) as a function of the extent of public coverage for the case of measles. The estimates of the epidemiological parameters for measles were obtained from Roy Anderson and Robert May (1991). The incubation period is estimated to be around ten days, which reduces to the year-adjusted duration \( 1/w = 71065 \). The life expectancy is taken to be 70 years, so that \( 1/m = 70 \). For measles, infection in developed countries such as the United States tends to result in morbidity rather than mortality, so that including nonzero mortality due to infection does not affect the results. The transmission parameter \( \beta \) was estimated using the fraction of susceptibles, estimated to be around 7 percent for measles.

The figure displays the total vaccine demand for the case of no prevalence response of demand, \( \theta_i = 0 \); prevalence-responsive demand, \( \theta_i > 0 \); and highly prevalence-responsive demand, \( \theta_i \gg 0 \). The slope of the total demand curve corresponds to the degree to which public coverage crowds out private coverage. The slope is unity in the case when there is no prevalence response, and hence no crowding out, and is close to zero in the highly prevalence-responsive case, when there is almost full crowding out. Only when the public coverage is above \( 1 - S \), about 93 percent in the measles case, does eradication occur, since then \( R_0 \ll 1 \). The figure thus illustrates the limits of mandatory vaccination in achieving eradication under a prevalence-responsive demand.

III. The Dynamic Welfare Effects of Eradication

Although the focus of this paper has been on the positive aspects of disease eradication, its normative desirability is important to address as well. In a static model analysis, it is clear that a full compulsory vaccination policy is dominated by a free-market solution: static externalities cannot justify an eradication program by themselves. However, this result does not hold when dynamic externalities are taken into account, since the most important welfare gain of an eradication program is that it eliminates costly prevention efforts in the future. A lower bound \( B \) on the willingness to pay for eradication by future generations is given by the net-present value of these eliminated prevention expenditures, as in

\[ B = \int_{t=0}^{\infty} \bar{D}(p) e^{-\delta t} dt = \frac{p \bar{D}(p) e^{-\delta t}}{\delta} \]

On the other hand, an upper bound \( \bar{B} \) on the payment that has to be made to current generations through full subsidies made to everyone (to maintain demand throughout the eradication program) is given by

\[ \bar{B} = \int_{t=0}^{T} p e^{-\delta t} dt = \frac{p(1 - e^{-\delta T})}{\delta} \]

The upper bound must include subsidies to everyone, since as prevalence goes to zero, no one vaccinates. The prevention expenditures avoided, therefore, are larger than the required subsidies whenever

\[ B \geq \bar{B} \iff \bar{D}(p) \geq e^{\delta T} - 1. \]

It follows that the interest rate \( \delta \) is a central factor determining the desirability of eradication. If the interest rate is zero, then eradication always is desirable for all demand functions, since the value of the future prevention it eliminates always is larger than the current cost of the eradication program. On the other hand, if there is full discounting, eradication always is dominated by the free-choice equilibrium. Also, a longer eradication program naturally makes this condition less likely to hold, as does a low level of demand caused by high prices or features of the infection that makes steady-state demand low.

It is useful to compare this to the profit motives of a subsidized monopolist. The important point here is that the monopolist does not obtain any profits from individuals born after eradication has been achieved (i.e., after \( T \)).
The rate of interest, therefore, has opposing effects on the profitability and welfare gain of eradication: less discounting makes the monopolist less likely to eradicate, although it is more likely to improve welfare. Thus, a deficit-financed eradication program, which spends beyond tax revenues during its operation but recoups the deficit in future generations, may improve welfare when discounting is at current market rates. This is so because such a program would allow for the intergenerational transfers that are necessary to pay current generations to overvaccinate for the benefit of future generations, which although benefit, do not compensate the vaccine manufacturers.

APPENDIX

Individual Vaccine Demand.—To show the individual behavior that leads to prevalence-dependent vaccine demand, let \( u(h) \) and \( V(h) \) denote the instantaneous utility function and value function in health state \( h = s, i, r, d \), representing susceptibility, infection, recovery through immunity, and death. We assume \( u(s) = u(r) < u(i) \), that is, being susceptible or immune gives the same instantaneous utility, which is smaller than that under infection. Without loss of generality, normalize the value of death to zero,

\[
u(d) = 0.
\]

The value of immunity satisfies

\[
(A1) \quad V(r) = u(r) + (1 - m)V(r) + mV(d),
\]

and hence \( V(r) = (1/m)u(r) \), which is the expected duration of immunity times the instantaneous utility of immunity. The value of infection similarly satisfies

\[
(A2) \quad V(i) = \frac{1}{w + m}[u(i) + wV(r)],
\]

which is the average duration of the infection times the expected utility when infected. If the cost of vaccination at time \( t \) is a positive and strictly increasing function of the price \( C(p_t) \) then, to retain tractability, we assume that the agent has the opportunity to vaccinate only at birth. Vaccination gives utility \( V(r) - C(p) \), as compared with no vaccination, which gives utility \( V(s) = V(r) - \pi(I')\Delta \), with

\[
(A3) \quad \pi(I') = 1
\]

\[
- m \int_{\Delta}^{\infty} \exp\left(- \int_{\beta I_s + m}^{\infty} du\right) ds
\]

The argument applies to allowing vaccination in any future period but is considerably more notation intense.
being the probability for a nonvaccinator to be infected in his lifetime; and \( \Delta = [u(s) - u(i)](1/w + m) \) is the cost of infection. An individual vaccinates if, and only if, \( C(p_i) \leq \pi(I') \Delta \). Notice that when prevalence \( I_t \) is constant at \( I \), this probability reduces to \( \pi(I')/\pi(1) = \pi(I')/\pi(1) \).

For any nonnegative price \( p_t > 0 \), the relative cost of vaccination remains uniformly positive for all \( x \), \( (C(x, p_t)/\pi(x)) = L_t > 0 \). Therefore, if a prevalence path \( I' \) is such that \( \pi(I') \leq L_t \), then for all \( x \), \( C(x, p_t) > \pi(I') \Delta(x) \), and therefore \( D(I', p_t) = 0 \), as required for prevalence dependence. Furthermore, since \( \pi(I) \) increases with \( I \), the steady-state demand satisfies \( D_t \geq 0 \), and since \( C(x, p) \) increases with \( p \), it satisfies \( D_t \geq 0 \).

Existence and Properties of the Steady State.—The steady-state conditions are

\[
\begin{align*}
\frac{dS_t}{dt} &= 0 : b[1 - D(I, p)] \\
\frac{dI_t}{dt} &= 0 : \beta S - mS = 0 \\
\frac{dR_t}{dt} &= 0 : bD(I, p) + wI - mR = 0.
\end{align*}
\]

With no disease-induced mortality, and with a birth rate equal to the death rate \( b = m \), the total population size is constant, and the third condition is redundant. The second steady-state condition for prevalence implies that either there are no infections, \( I = 0 \), or that there is a positive steady-state prevalence \( I > 0 \), and that the susceptible fraction satisfies \( S = 1/R_t = (w + m)/\beta \). This requires that \( R_t > 1 \), in order for there to be a positive steady-state prevalence. Inserting the steady-state level of susceptibles \( S \) into the first steady-state condition defines the prevalence as an implicit function of price through the equation \( F(I, p) = 0 \), with

\[
F(I, p) = b[1 - D(I, p)] - \beta IS - mS.
\]

The condition \( D(0, p) < 1 - S \) implies \( F(0, p) > 0 \). Furthermore, when \( b = m \), \( F(1, p) < 0 \). When \( D_t \geq 0 \), we have that \( F_t = -bD_t - \beta S \leq -\beta S < 0 \); therefore, for any price \( p \) there exists a unique prevalence level \( I(p) > 0 \), such that \( F(I(p), p) = 0 \). And since \( F_t = -bD_t \geq 0 \), the implicit function theorem implies

\[
I_t = \frac{-F_t}{F_p} = -\frac{1}{D_t + \frac{w + m}{m}}D_t \geq 0,
\]

as required for prevalence to rise with price. Furthermore, for all \( p \),

\[
F(I(p), p) = 0 = F(0, p) + \int_0^{I(p)} F_t dI.
\]

If an alternative demand function \( D' \) is such that \( D_t' \) is uniformly larger that \( D_t \), then

\[
\beta SI(p) = F(0, p) - b \int_0^{I(p)} D_t dI
\]

\[
> F(0, p) - b \int_0^{I(p)} D_t' dI
\]

\[
= \beta SI'(p).
\]

The steady state associated with \( D' \) is lower. Finally, this steady state is locally stable: computation of the Jacobian matrix \( J \) of the system, evaluated at the steady state, leads to

\[
J = \begin{bmatrix} -\beta I(p) - m - mD_t - \beta S \\ \beta I(p) \end{bmatrix}.
\]
The product of the two eigenvalues of this matrix is positive, and their sum is negative. Therefore, they both have a negative real part, which proves the stability of the steady state.

REFERENCES


