ASPECTS OF PUBLIC EXPENDITURE THEORIES*

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ECONOMIC theorists have done work of high quality and great quantity in the field of taxation. Public expenditure seems to have been relatively neglected. To illustrate this, let me turn to Professor Pigou. I do so with some diffidence, remembering what Ralph Waldo Emerson said to Oliver Wendell Holmes when Holmes showed him a youthful criticism of Plato. "When you strike at a King," Emerson said, "be sure you kill him."

I have no wish to assassinate Professor Pigou. Nor even to criticize him. But immortality does have its price: if one writes an outstanding treatise such as Pigou's A Study in Public Finance, one must expect other men to swarm about it, picking a nugget here and probing for a weakness there.

Of a book of some 285 pages, Pigou devotes most attention to taxes. At least 200 pages to taxes; of the rest, most are concerned with fiscal policy and its impact on the business cycle. What about the pure theory of public expenditure? I can find barely half a dozen pages devoted to the heart of this matter—specifically, pages 30-34. And even if we widen the category—to include Pigou's definitions of transfer and exhaustive expenditure and his discussion of pricing of state-operated public utilities—we still cannot bring the total of pages much beyond twenty.

Now it may be that this ratio of 200 on taxes to 20 on expenditure is the proper one. Perhaps there is really nothing much to say about expenditure, and so heavily overbalanced a page budget may be truly optimal. On the other hand, we must admit that fashion has a great influence in economics, which suggests that we ought periodically to survey the neglected areas of theory to make sure that they do deserve to be left in their underdeveloped and backward states.

I have previously published (this REVIEW, xxxvi, November 1954, 387-89) some thoughts on public expenditure theory; and in order to widen the discussion among economic theorists, I later gave a non-mathematical exposition (ibid., xxxvii, November 1955, 350-56). I do not propose here to give a detailed review of these theories. Rather, I'd like to think aloud about some of the difficulties with expenditure theory and with political decision-making. On these subjects, Richard Musgrave and Julius Margolis have done outstanding research and I must confess my obligation to them for much friendly counsel.

I

Let me first take a fresh look at the nature of government and of public finance from a purely analytical viewpoint. I must give warning: the result will be rather like a New Yorker's map of the United States, in which vast areas of the country are compressed into almost nothing and certain places—like Hollywood, Cape Cod, and Times Square—are blown up far beyond their true proportions.

Similarly, I shall commit all the sins of those bad historians and anthropologists who recreate the history of the human race according to their a priori conceptions of the moment. To keep from getting caught, I'll imagine a planet rather like the earth.

Once upon a time men on this planet were all alike and very scarce. Each family hunted and fished its symmetrical acres; and each ended with the same production and real income.

Then men turned to cultivating the soil and domesticating animals. This left even more of the globe vacant, but did not disturb the symmetry of family incomes.

But finally population grew so big that the best free land was all occupied. Now there was a struggle for elbow room. According to the scenario as I choose to write it, the struggle was a gentlemanly one. But men did have to face the fact that recognizing squatter's rights and respecting laissez-faire did result in differences of real incomes among families.

Optimal transfer expenditure. Here then for the first time, government was introduced on
this planet. A comprehensive program of redistributing income so as to achieve a maximum of the community’s social welfare function was introduced. The budget was balanced at a non-zero level: taxes were raised in a non-distorting lump-sum fashion, and transfer expenditure was allocated among families so as to achieve the marginal conditions necessary to maximize the defined social welfare function.

Now here on earth, things don’t seem to have worked out exactly according to such a timetable. In fact, look at Adam Smith’s 1776 discussion of the three duties of government — protection against external aggressors, maintenance of order at home, and erecting those public institutions and works “which though they may be in the highest degree advantageous to a great society, could never repay the expense to any individual.” We could interpret the last of these in so broad and tautological a way as to be compatible with anything. But if we stick to a narrower non-empty interpretation, it would appear that our planet began with redistributive governmental functions that Smith had not even dreamed of and which would most surprise him if he were to come to life and revisit any modern nation.

Now why do I describe so bizarre a model? It is to underline this theoretical point: Given a social welfare function, and given the absence of all technological and taste externalities, and given universal constant returns to scale, there would be needed only one type of public policy — redistributive transfers. (Under some ethical assumptions, these might be from poor to rich rather than rich to poor; but only by chance alone would zero redistributions maximize a specified social welfare function that depends solely on real incomes.)

Minimal collective expenditure. But what about the neglected exhaustive elements of public expenditure that even the most thorough-going laissez-faire economy will want to make — e.g., courts of justice to enforce contracts or any of the other items under Smith’s first two duties? Later I shall review a possible theory of such expenditure. But first let me mention why the problem of financing such expenditures is, so long as they remain small, secondary to that of transfers.

Even on other planets, perfectionist lump-sum taxes are rarely feasible. We tax the objects that we can feasibly tax. And this must introduce deadweight-theoretically-avoidable tax burdens in addition to the unavoidable real burden involved in having to use resources for public purposes. (This doesn’t mean the public services aren’t worth their costs; on well-run planets, they are.)

Years ago, when studying this problem, I encountered what was to me a surprising fact. It turns out that, so long as exhaustive expenditure is “small,” the deadweight burden is “negligible” no matter what system of taxation is used. Only in the second approximation, so to speak, does it matter what tax structure we use to “cover” the needed program. At least this would be the case if incomes were already distributed optimally. If, as is more likely, incomes are distributed prior to taxation in a non-optimal manner (not as determined by me but as determined by the relevant social welfare function), then the manner of taxing is very important even at the first level of approximation; and it is the interpersonal distributive elements that are all important in defining an optimal tax structure.

It is because of this conclusion that my planet had to start out with transfer taxation. As I have said, this result seemed odd to me at first; but having been led there by the invisible hand of mathematical logic, I was forced to draw my map in this way.

Sizable exhaustive public expenditure. Once we admit the possibility of public collective services on our planet, we have to face the possibility that they will be large rather than small; and in any case they will be finite rather than zero or infinitesimal. So we do need an analysis of their logical nature.

We can approach this indirectly. What is our theory of non-public expenditure? So long as goods are producible at constant returns to scale and so long as each person’s consumption of a good is measurably distinct from any other person’s, the perfect-competition model of markets can be used as an optimal social computing device. If we deny constant returns to scale — and technology on this or any other planet may make this denial mandatory in many areas—an opening wedge for an alternative kind of social allocation arises. And if we deny that every good’s consumption is purely individualistic, in-
stead insisting on strong "external effects," we will have still another reason why the ordinary private marketing calculus must be non-optimal.1

To handle one difficulty at a time, let's keep to a strict assumption of constant returns to scale in all production. But let's introduce important externals ("neighborhood" effects, etc.) into the consumption sphere. Thus, the battleship that protects your rights and investments also protects mine.

I don't suppose that anyone, upon reflection, would try to build up a theory of public expenditure without bringing in some kind of externality. Yet it is surprising that Pigou, who above all welfare economists has reminded us of external diseconomies of the smoke nuisance type, should in his brief discussion of expenditure theory have left this externality element almost completely implicit.

Now remembering that we theorists like to work with extreme polar cases, what is the natural model to formulate so as to give strongest emphasis to external effects? I have long thought that this is best brought out by the following model.

Assume that some goods, like bread, are privately consumed: this means that the total of bread can be written as the sum of the bread consumptions of each separate individual. But along with such purely private goods, assume public goods — like national defense — which simultaneously enter into many persons' indifference curves. Then assuming no transcendental group mind, but only a set of individual tastes and an ethical social welfare function dependent upon these tastes and ranking them in order of deservingness, we can prove that the perfect-competition market model will not work optimally. We can prove that there exists an inefficient configuration from which all men can be made better off, and a frontier of efficient points from which no universally advantageous movements are possible; of all the infinity of such efficient points, a socially best one is definable in terms of a specified normative welfare function.

It is this model that I explored in the two cited papers. And it is also this model that Sax, Wicksell, Lindahl, Musgrave, Bowen, and other economists of the last 75 years had considered under the "voluntary exchange theory of public finance" name or some other. The principle conclusions of this analysis seem to be the following:

1. Efficient, inefficient, and socially optimal configurations can be theoretically defined: a point on the efficiency frontier requires equality between the vertically-added marginal rates of substitution of all men for the public and private goods; and the best of such points requires lump-sum redistributions of the transferable private goods until they have equal marginal social significance.

2. Although the optimum is definable, rational people will not, if left to themselves, be led by an invisible hand to the bliss point. On the contrary, it will pay for each rational man to dissemble, trying to mask his preference for the public goods and to engage in other game-strategy maneuvers which, when all do them, will necessarily involve deadweight loss to society.

Having called attention to the nature of the difficulty, I do not wish to be too pessimistic. After all, the world's work does somehow get done. And to say that market mechanisms are non-optimal, and that there are difficulties with most political decision processes, does not imply that we can never find new mechanisms of a better sort. (Example: skillful use of the symmetry that prevails between individuals may enable us to find optimal computing algorithms. Example: Interrogate people for their tastes with respect to public goods in such large homogeneous groups as to give each respondent the feeling that his answer can be a "true" one without costing him anything extra.)

Decreasing-cost phenomena. Once people have understood the above model, they are likely to object to its unrealism. Thus, Drs. Stephen Enke and Julius Margolis have both pointed out that many, if not all, government expenditures can be qualitatively varied so as to confer more

1 There are still other basic reasons for governmental action or interferences: e.g., "paternalistic" dissatisfaction by the electorate with the effective tastes that they will display in their day-to-day market preference acts — leading to public policies in the field of education, capital formation, etc.; exercise of economic entrepreneurship and decision-making by public officials; and many more.
benefit on one man at the expense of another man. This raises the question whether we cannot bring back the market pricing mechanism, charging fees for public services and letting their quantity and quality be determined by money voting of the supply and demand type.

Certainly, it should be possible for the theorist to go beyond the polar cases of (1) pure private goods and (2) pure public goods to (3) some kind of a mixed model which takes account of all external, indirect, joint-consumption effects. I shall not write down such a mathematical model. But if I did do so, would we not find—as Pigou and Sidgwick so long ago warned us—true of all external economies and diseconomies—that the social optimum could not be achieved without somebody’s taking into account all direct and indirect utilities and costs in all social decisions?

Now in connection with running a particular railroad, highway, or concert, we might find just the right conditions of scarcity of space and of independence of consumptions so that ordinary market pricing could lead to the optimum. In such a case, we can really reduce matters to our first category of purely private goods, and self-policing perfect competition might be an optimal social signalling and computing device. However, generally, a mixed model that refuses to fall in my polar case of a pure public good will not thereby obligingly go into the other polar case of a pure private good. The mixed case has elements of both in it. And while we cannot by pure logic alone deduce that the intermediate case must qualitatively be a blend of the properties of the two poles, we can by logic know that ordinary pricing will be non-optimal unless it happens to be able to pick up each indirect external marginal utility.

Here is a contemporary instance. The Federal Communications Commission is now trying to make up its mind about permitting subscription television. You might think that the case where a program comes over the air and is available for any set owner to tune in on is a perfect example of my public good. And in a way it is. But you would be wrong to think that the essence of the phenomenon is inherent in the fact that the broadcaster is not able to refuse the service to whatever individuals he pleases. For in this case, by use of unscramblers, it is technically possible to limit the consumptions of a particular broadcast to any specified group of individuals. You might, therefore, be tempted to say: A descrambler enables us to convert a public good into a private good; and by permitting its use, we can sidestep the vexing problems of collective expenditure, instead relying on the free pricing mechanism.

Such an argument would be wrong. Being able to limit a public good’s consumption does not make it a true-blue private good. For what, after all, are the true marginal costs of having one extra family tune in on the program? They are literally zero. Why then prevent any family which would receive positive pleasure from tuning in on the program from doing so?

Upon reflection, you will realize that our well-known optimum principle that goods should be priced at their marginal costs would not be realized in the case of subscription broadcasting. Why not? In the deepest sense because this is, by its nature, not a case of constant returns to scale. It is a case of general decreasing costs. So long as increasing returns prevail in the actual range of consumption, we know that perfect competition will not be self-preserving and market behavior is unlikely to be optimal.

The case of decreasing costs may be empirically very important. Certainly, when you try to analyze why public utilities are public utilities and why certain activities (like railroads, water supply, electricity, and postoffices) may fall into either the category of public or private enterprise, you will usually find that some significant deviation from strict constant returns to scale is involved. I cannot then be completely satisfied with Pigou’s statement:

> These are not problems of Public Finance, as I understand that term. I do not propose, therefore, to discuss at all the question over what classes of enterprise it is desirable that public operation should be extended, but to proceed on the assumption that this is already determined. (Page 24.)

Considerations concerning waste thus enable us to say, with regard to several classes of goods and services [primarily those which do not have an inelastic demand], that, if the government decides to provide them, it should finance their provision by fees. (Pages 27–28.)

It is precisely in such cases that uniform average cost pricing will sin against the rule that prices should equal marginal costs. As Hotelling has insisted, there is here a *prima facie* case for
government subsidy. To argue, as some economists have done, that the government budget is already so loaded with necessary expenditure as to make it undesirable for it to have to take on such subsidy expenditures, is to miss the point I am trying to make. This is one of the needed functions of government, and in making compromises because of fiscal necessities, there is no a priori reason why this function should be particularly neglected.

There is a related significant point that needs stressing. It is not enough in the decreasing cost case to come closer to marginal cost pricing in the Lerner-Lange manner, making up the deficits by general taxation. As soon as decreasing cost and diversity of product appear, we have the difficult non-local "total conditions" to determine what finite mix of product is optimal. This involves a terrible social computation problem: we must scan the almost infinite number of possible products and select the best configuration; we cannot feel our way to the optimum but must make judgment at a distance to determine the optimum optimorum.

All this is familiar. But what I have to point out are the complications that arise when there are two or more people on the planet. I like my cider sour; you like it sweet. With constant returns we could both get what we want, or at least what we deserve. But with initial indivisibilities or other forms of increasing returns, what I get will depend on what you get. (This is true even if we pay in the form of fees the marginal costs of our separate consumption.)

Now, how can society decide on the product mix which will maximize a specified social welfare function? It must weigh in all the different individuals' utilities from each decision. And this is a problem that is analytically almost exactly like my model of public expenditure.

Given the individual indifference curves and the social welfare function compromising them, one could define the theoretical optimum. (In practice, finding the solution might be very tedious.) But now try to devise a system of "benefit taxation" that will in some sense make people pay for what they get—either because justice or equity requires this, or, more subtly, because the necessity of having to make such payments is thought to be a way of helping to determine the proper place for society to arrive in the end. Instantly, you will discover that the same game-theory reasons that compel rational men to hide their desires for public goods will be motivating them to hide their consumers' surpluses from different product configurations.

II

Once again, in contemplating the dilemmas that most forms of political voting involve, we are reminded of the beautiful and special simplicities of the laissez-faire model. But, alas, the difficulties are those of the real world. And it would be quite illogical to conclude from all this that men and technology should be different, should be such as to make the competitive game all-sufficient. That would be as silly as to say that we should all love sawdust because its production is so beautiful.

Conclusion. Unfortunately, I have only gotten my planet started. Time hasn't permitted me to do more than describe its transfer expenditures, to relate them to the financing of small public services, and to formulate some of the analytic difficulties with a theory of public services. Though my model of pure public goods has turned out to be an unrealistic polar case, it turns out that almost all deviations from constant returns to scale and almost all externalities must inevitably involve some of the same analytic properties and dilemmas of my polar case.

We must leave to other times and other stars the exploration of those momentous coalitions of decision-making that are part of the essence of the political process. To the theorist, the theory of public finance is but part of the general theory of government. And at this frontier, the easy formulas of classical economics no longer light our way.
Appendix: Strotz and Tiebout Discussions

1. Distributional aspects of public goods. I should like to comment briefly on two papers that have grown out of the earlier discussion. In the present issue Professor Robert H. Strotz has pointed out a formal implication of my original equations (1) and (2): they define a Pareto optimality frontier

\[ u^i = f(u^2, u^3, ..., u^j); \quad \text{(A)} \]

and each point on that frontier will generally determine a set of all public goods

\[ X_{n+j} = g(u^2, u^3, ..., u^i) \quad (j = 1, 2, ..., m). \quad \text{(B)} \]

Now under what conditions can the left-hand variables in (B) be regarded as independent variables? If \( s - 1 = m \) and the Jacobian matrix \([g^i_j]\) is well-behaved, (B) can be inverted. Or if \( s - 1 > m \) and the Jacobian \([g^i_j]\) is of rank \( m \), \( m \) of (B)’s right-hand \( u \)’s can be solved for in terms of the public goods \((X_{n+1},...,X_{n+m})\) and the remaining right-hand \( u \)’s. So the Strotz conclusion follows: Any public good configuration is optimal if only the “distribution of income” is such as to get us to a point on the Pareto-frontier compatible with that public good configuration.\(^8\)

In view of the modern trend to regard mere Pareto-optimality or efficiency as incomplete necessary conditions, what follows from the above conclusion? To me, this.

It is wrong to make, as some have made, a sharp separation between correct public-good decisions and correct redistributional-taxation decisions. Changing public goods does materially affect the distribution of income and all decisions have to be made simultaneously.

As Professor Strotz says, there is no disagreement between our analyses.

2. Local finance and the mathematics of marriage. A second paper of interest in the present connection is that of Professor Charles M. Tiebout.\(^4\) He argues that the public expenditure theory simplifies itself at the local level — as people spontaneously join in forming homogeneous communities which will legislate what each (and all) want in the way of collective goods.

This attempted solution fits in under one or another of the “symmetry” principles that I had referred to. That it goes some way toward solving the problem, few would doubt. As a solution, though, it raises a number of serious questions.

Thus, when you study in detail a supposedly homogeneous suburb, you find it riddled with conflicting desires. The old, with grown-up children, oppose the desire of the young for more school expenditure. And so it goes. It avails little for one group to say to another: “If you don’t like it here, go back where you came from.” Ours is a fluid society, with little respect paid to hard-to-identify charter members. People want to “improve” their community, not abdicate from it.

Secondly, people often like heterogeneity even though it involves conflict. The old don’t want to live in homogeneous ghettos with their own kind, and the same goes for many other groups. In an interdependent world, one man’s privacy is another man’s condemnation to loneliness.

Thirdly, there is the political and ethical question whether groups of like-minded individuals shall be “free” to “run out” on their social responsibilities and go off by themselves. At the national level, society respects no such freedom: e.g., migration control, compulsary taxation, etc.

A simple mathematical model will illustrate a few of the intricacies of the problem. If a group of men and women have each a preference rating for members of the opposite sex, who will end up marrying whom? This assignment problem — which is stated in biological terms only for concreteness — is also faced by colleges and students choosing each other, by clubs and fraternities, etc. In real life, it is solved by dynamic reconnoiter, contact, proposal, refusal or acceptance — in short, by general trial and error, which is not guaranteed to represent any optimum.

Consider the trivial case of 2 boys A and B and two girls 1 and 2. Each boy has an ordinal preference rating of the girls, which in this simple case must be either the permutations \((1, 2)\) or \((2, 1)\). Each girl’s rating of the boys must be \((A, B)\) or \((B, A)\).

Now what are the possible preference configurations? In this simple case, they are essentially only the following (where the first row lists people, with their choices shown in the columns below them):

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<tr>
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<tr>
<td>B</td>
<td>A</td>
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or

\[ A \quad B \quad 2 \quad 1 \]
Of these four cases, the first fits Tiebout’s attempt best. All the guinea pigs are in agreement: 1 and A want to marry; so do 2 and B; all get their first choices. The solution (1, A; 2, B) is Pareto-optimal; being the only Pareto-optimal solution, it is also Bergson-optimal, maximizing any social welfare function that respects individual’s tastes.

But now turn to the last case. A and 1 are preferred by all of the other sex. If we give persons the “property right” to form bilateral compacts, the favored ones will presumably marry each other with (1, A; 2, B) the resulting equilibrium.

However, given a social welfare function which respects tastes, this outcome is not necessarily optimal. The other possible outcome (2, A; 1, B) might be “ethically better” (e.g., where 2 has a “great” preference in favor of A but 1 is “almost” indifferent, and 2 is “ethically deserving” of great social respect). Or we can put the matter a little differently: (2, A; 1, B) is just as Pareto-optimal as is (1, A; 2, B). When you leave the former and go to the latter, you make two people happier and two people unhappier.

In the second case, there is likewise no unique Pareto-optimal point. Left to themselves with certain “freedoms” and “property rights” to make bilateral collusions, probably 1 and A will marry, ignoring 2 and B. And B will be glad. But 2 will not be glad, showing that the configuration (2, A; 1, B) is also Pareto-optimal. Of course, if you used a crude majority vote rule, (1, A; 2, B) would be the winning position. But — as Arrow, Black, and others have shown — majority-rule devices are subject to many intransitivities and drawbacks.

Finally, the third case is like the last in that both outcomes are Pareto-optimal and in going from one to the other you sadden two people and gladden two. Whether the girls or boys are to be made glad cannot be decided except in terms of a given determinate social welfare function.

I conclude from all this that there remain many important analytical problems of public-good determination that still need investigation at every level of government.